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EVALUATION FOR SOIL OF CRACK NET CONNECTEDNESS AND CRITICAL STRESS-INTENSITY FACTOR

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A b s t r a c t. Based on data reflecting size effect of soil clod strength, and on theoretical computation of clod strength, an approach was suggested for evaluation of crack net connectedness and critical stress-intensity factor for a given soil. For validation of the suggested approach published soil clod strength data were used. The results show the possibility of computing clod size distribution after tillage, which can then be used as an initial state for the next processes, e.g., erosion, and also enables computing internal clod cracking and magnitude of related parameters.

K e y w o r d s: soil mechanics, clods, cracking, strength, scale effect

INTRODUCTION

Cloddiness of air-dry soil after tillage is an important characterisation of the initial soil state for following processes, e.g., erosion. In [5] a mathematical model predicting soil cloddiness formation after tillage was suggested and validated on published data. Soil cloddiness is determined by the average cracking (average number of cracks of a size x in a volume of the same size) [2,3]:

$$I(x) = ln\ (K^* + 1)\ c\ (x\ /\ d)^4\ \exp\ (-x\ /\ d) \qquad (1)$$

where K^* is critical value of mean distance between microcracks (in units of their characteristic size [9,10], which is determined by soil microstructure). Here, the mean distance is understood as reciprocal of cube root of mean volume concentration of microcracks [9,10]. For rocks and soils K^* 5 [9,10]. Parameter d is the mean spatial distance between cracks, which occur chaotically in the volume under consideration. In frames of the model $d=x_m/4$, where x_m is maximal clod size in the given clod size distribution. The dimensionless factor c, which must not surpass one, describes the extent of crack coalescence in a given crack net [4]. Hereafter the factor is referred to as crack net connectedness (CNC). The CNC determines the volume share f_m of the clods, delineated, or nearly delineated, by the cracks, which form the given clod size distribution:

$$f_m = 1 - \exp\ (-I\ (x_m)) = 1 - \exp\ (-\ 8.4\ c)\ . \qquad (2)$$

The remaining part of a soil consists of very large clods (huge-clods), which are essentially conglomerates of clods, which are not completely divided in smaller sizes.

For practical application of the model [5] it is necessary to know the CNC-value for a given soil. The main purpose of the article presented is to suggest an approach to estimate the CNC-value, on the basis of clod strength data determined by use of compression fracture tests. For a practical demonstration and validation of the suggested approach, the data of Hadas and Wolf [6] were used.

Moreover the approach under consideration enables attainment of an appraisal for mode I crack tip critical stress-intensity factor K_{Ic} (according to fracture mechanics the magnitude determines crack development resistance of a brittle material by normal rupture and has value characteristic of the material) suitable for air-dry soil. The K_{Ic}-value can be used in describing the future crack development in clods, as a result of their swelling and shrinkage. The critical stress-intensity factor is proportional to the square root of the specific surface energy, estimation of which for air-dry soils has been partially carried out based on clod strength data of two soils [6]. However, the analysis in [6] used the Griffith' expression for clod strength. This expression does not depend on the body's size, that contains the crack, and suits only cases where the clod size is large enough in comparison with crack size. In the following presentation this limitation has been removed.

THEORY

For computing clod strength the concepts of Chertkov's work [4] are used. A soil clod is characterised by one size x, and the longest crack inside it is characterised by size x', and $x>x'$. The number of cracks in a clod is large, and their distribution in the clod's volume is supposed to be homogeneous, and their orientation distribution is taken to be isotropic. Then it may be assumed that by tensioning the clod there will exist a maximum crack, which is normal to tensile stresses applied. Two cases are possible: a) a crack is an edge crack (sign '+'); and b) a crack is an internal crack within the clod (sign '-'). The realisation probability for the first case is estimated by the following function:

$$\varphi_+ (x'/x) = x'/x \tag{3}$$

and for the second case by the function:

$$\varphi_- (x'/x) = 1 - x'/x . \tag{4}$$

The tensile strength for the first case is approximated by applying an expression corresponding to the tensioning of a strip of a width x with an edge crack of length x', which is normal to the external tensile force (and to the strip) [1], and is given by:

$$\sigma_+ (x'/x) = K_{Ic} \ (\pi \ x')^{-1/2} \ (1 - x'/x) \,/\, (1.11 + 5 \ (x'/x)^4). \tag{5}$$

For second case (neglecting any effect of crack position in a clod) the tensile strength is approximated by an expression corresponding to the tensioning of a strip of width x with a central crack of length x', which is normal to the external tensile force (and to the strip) [7] and is given by:

$$\sigma_- (x'/x) = K_{Ic} \ (\pi \ x')^{-1/2} \ (2\cos ((\pi/2) \ (x'/x)))^{1/2}. \tag{6}$$

Averaging the strength on the basis of possible positions of the maximum crack in the clod leads to the replacement of the σ_+ and σ_- functions, Eqs. (5) and (6) respectively, by:

$$\sigma(x', x) = \varphi_+ (x'/x) \ \sigma_+ (x', x) + \varphi_- (x'/x) \ \sigma_- (x', x) . \tag{7}$$

The strength of a clod with the size x, which is additionally averaged on the basis of the possible sizes of the maximal crack being present in it, is determined by the expression:

$$\sigma(x) = \int_0^x \sigma(x', x) \, P(x', x) \, dx' \tag{8}$$

where $P(x',x)dx'$ is the probability that the largest crack with a size between x' and $x' + dx'$ of any possible orientation and position exists in a clod of size x. Using Eq. (1) one obtains the following expression for $P(x', x)$ (see [4]):

$$P(x',x) = \frac{(cx^3/d^4)\, e^{-x'/d} \exp\left(-\left(e^{-x'/d} - e^{-x/d}\right) cx^3/d^3\right)}{1 - \exp\left(-\left(1 - e^{-x/d}\right) cx^3 / d^3\right)}. \tag{9}$$

According to the Eqs. (3)-(9) one can then write down the expression for the tensile strength of soil clods of size x:

$$\sigma(x) = K_{Ic}\ d^{-1/2}\ S\,(c, x/d) \qquad (10)$$

where dimensionless S-function does not depend on K_{Ic}. The scale effect in the strength-clod size relation, which depends for a given d-value only on the crack net connectedness, is given by Eq. (11), where x_o is a reference clod size:

$$\sigma(x)\,/\,\sigma(x_o) = S\,(c, x/d)\,/\,S\,(c, x_o/d). \quad (11)$$

The CNC-value for a given soil may be estimated by matching experimental data of strength scale effect (that is the left side of Eq. (11) as function of x for a certain x_o) with a set of theoretical dependencies, computed for a number of CNC - values (that is the right side of Eq. (11) as function of x for the same x_o-value). Herewith one ought to consider and handle the occurrence of large scatter of strength data. For this comparison one may use the compressive strength data too, because the strength scale effect depends only very slightly on the strength test type if at all.

The critical stress-intensity factor is a characteristic of a given soil. K_{Ic}-values should not depend on clod size, this point is essential for the transition from Eq. (10) to Eq. (11). The problem of the critical stress intensity factor independence on clod size is discussed in the next paragraph. K_{Ic}-values can be estimated by using expression (10) for the same data after determining the CNC-values and for a known d-value. Herewith it is assumed that tensile and compressive strengths are proportional each other.

RESULTS: ESTIMATION OF CNC- AND K_{Ic}-VALUES

For validation and demonstration of the suggested approach use of data of Hadas and Wolf [6] (Fig. 1a, b) is made. In their work compressive strength σ_r (by definition this is stress at rupture of soil aggregate) was measured following the method reported by Rogowski *et al.* [18]. There the strength data for two soil types were obtained. Three cases were examined - a nonswelling sandy loam soil (soil 1) in a virgin state and after cultivation for 27 years; and a swelling type clay soil (soil 2) after 45 years of cultivation (detailed physical and chemical properties for both soils are given by Hadas and Wolf [6]). In their work statistical correlations of the form $y = a\,x^b$ were established between the compressive strength and the clod size (Fig. 1a, b; solid curves), of both soils.

In the present work upper and lower boundaries of the experimental data field were plotted (Fig. 1a, b; dotted curve) for each of the three cases. The values of the scale effect M_m were computed on the basis of the measured strength (marked by index 'm') as:

$$M_m = \sigma_r\,(x)\,/\sigma_r\,(x_o) \qquad (12)$$

for a number of x - values and x_o = 1 mm. One obtains maximum values of M_m, by selecting the compressive stress at rupture $\sigma_r\,(x)$ from the upper dotted curve and $\sigma_r\,(x_o)$ from the lower dotted curve. Herewith possible $M_{m\,\max}$ -values larger than one are fictitious for obvious reasons and must be substituted by a value of $M_{m\,\max}$= 1. It is clear, that $M_{m\,\min}$ - values match the opposite choice: the compression stress at rupture $\sigma_r\,(x)$ from the lower dotted curve and $\sigma_r\,(x_o)$ from the upper dotted curve. For the three soil conditions under study the computed values of $M_{m\,\min}$ and $M_{m\,\max}$ are given in the Table 1 for a selected number of x - values and x_o = 1 mm. Thus for each soil condition the measured scale effect values occupy a certain strip in the 'clod size-scale effect' plane.

Before proceeding to the theoretical estimate of the scale effect it is necessary to remark about the choice of the mean clod size value. In the work of Hadas and Wolf [6] clods with a size range up to 200 mm were tested. For clods larger than a size of 80 mm one may approximate the strength by zero, considering the dispersion of its values. The

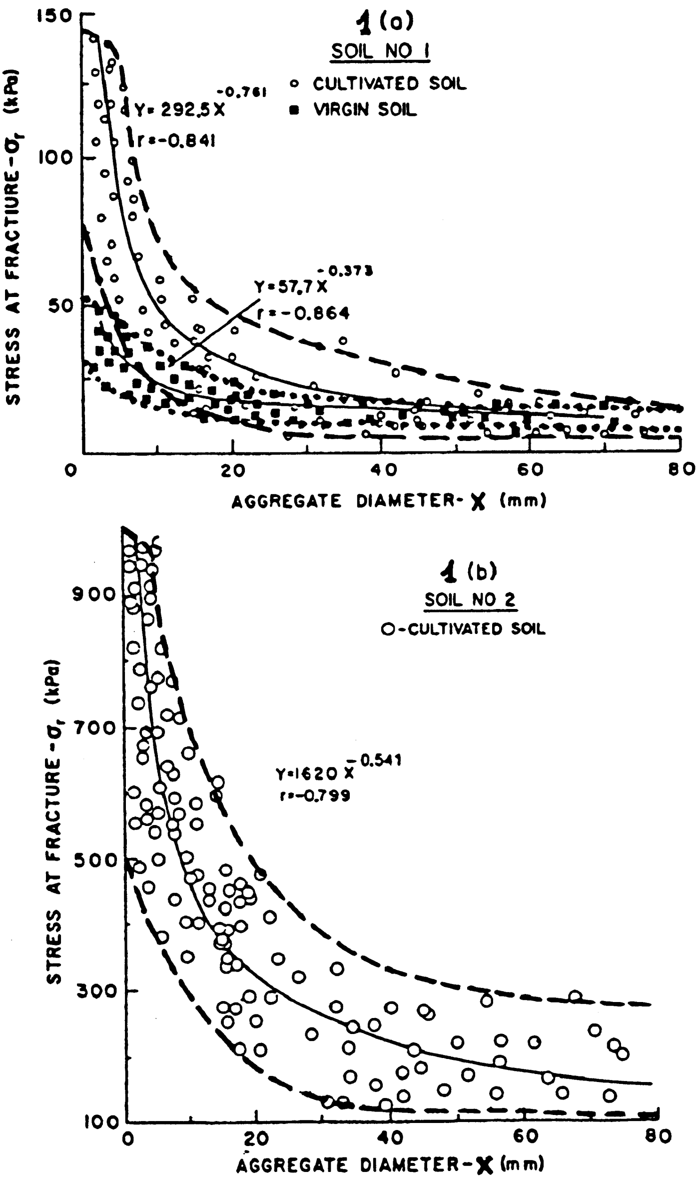

Fig.1. Stress at fracture (kPa) under compression as function of aggregate diameter (clod size, mm) (after Hadas and Wolf [6]).

decrease in strength of these clod sizes justifies the assumption that these clods are the very large clods mentioned at the beginning and are essentially conglomerates of not completely divided clods of smaller sizes. So, for the soils under study one may take the value x_m = 80 mm as the maximum clod size, determined by the cracking according to expression (1) and, hence, for the mean clod size the value $d = x_m/4 = 20$ mm.

The theoretical values of the strength scale effect (marked by index 't') given by:

$$M_t = S\,(c, x/d)\ /\ S\,(c, x_o/d) \qquad (13)$$

were computed according to Eqs. (3)-(11) for a

Table 1. Maximum (above) and minimum (below) values of measured strength scale effect as dependent on soil type, soil use and clod size (on basis of Hadas and Wolf'data [6])

Soil No.	Soil usage and duration of mechanized cultivation	Clod size (mm)						
		2	5	10	20	40	70	80
1	Virgin soil	1 0.50	1 0.35	1 0.28	0.86 0.20	0.64 0.15	0.55 0.15	0.55 0.15
	Cultivated for 27 yr	1 0.43	1 0.25	1 0.17	0.90 0.08	0.60 0.05	0.32 0.03	0.30 0.03
2	Cultivated for 45 yr	1 0.46	1 0.37	1 0.29	1 0.18	0.71 0.12	0.59 0.11	0.59 0.11

number of CNC- and x-values (x_o = 1mm, d = 20 mm) and presented in Table 2.

Comparison of the Tables 1 and 2 enables us to chose a certain value of the crack net connectedness for each soil type and soil usage under consideration. The CNC-value for each soil is uniquely determined as follows. The maximum value is chosen from the values that meet simultaneously following two conditions:

- computed scale effect values for a given CNC must be inside the measured values strip in the range of all clod sizes;
- computed scale effect values for a given CNC must be at least 1.5 or 2 times larger than the measured minimum scale effect in the clod size range larger 20 mm.

The first condition is evident. The second is related to the fact that, with the clod size range larger than 20 mm, the measured minimum scale effect is considerably less than the measured maximum scale effect (see Table 1). And finally the maximal CNC-value is chosen from a number of possible values following simple physical considerations. Cracking must result in complete stress relaxation in the clods. The larger the CNC-value, the more cracking (see formula (1)) and the more stress relaxation. The results of the CNC-value estimation for each soil condition are presented in the Table 3, together with the clod volume share for clod sizes less than 80 mm, according to Eq. (2). The complimentary values ($1 - f_m$) give the volume share of the very large clods.

The critical stress-intensity factor was estimated for each soil (by known CNC- and d-values) using the relationship given by Eq. (10):

$$K_{Ic} = \sigma(x)\ d^{1/2} / S\,(c, x/d)\,. \qquad (14)$$

For each soil the values of the dimensionless S-function was previously known for a number of x-values after determination of the CNC-value. During practical computation the tensile strength values were replaced by the proportional compressive strength values for the soils under consideration [6]:

$$\sigma(x) = \sigma_r(x) / 9.2\,. \qquad (15)$$

Choosing the strength values under compression for a given soil according to the upper

Table 2. Computed strength scale effect as dependent on crack net connectedness and clod size

Crack net connectedness	Clod size (mm)						
	2	5	10	20	40	70	80
0.01	0.72	0.47	0.36	0.29	0.26	0.23	0.22
0.035	0.72	0.47	0.36	0.29	0.24	0.18	0.15
0.07	0.72	0.47	0.36	0.29	0.23	0.12	0.09
0.1	0.72	0.47	0.36	0.29	0.22	0.09	0.07
0.2	0.72	0.47	0.36	0.28	0.18	0.05	0.04
0.3	0.72	0.47	0.36	0.28	0.14	0.03	0.03
0.6	0.72	0.47	0.36	0.26	0.08	0.02	0.01
1	0.72	0.47	0.35	0.24	0.04	0.01	0.01

dotted curve (Fig. 1) one may obtain the maximum values of the critical stress-intensity factor from Eqs. (14) and (15). Analogously, the lower dotted curve, for the strength values under compression, gives the minimum values of the critical stress-intensity factor. Knowing the maximum and the minimum values, one may estimate the mean K_{Ic}-value as their half-sum and the deviations from it, for a number of values of the clod size for each soil under consideration. The results of such computations are presented in Table 4. As expected for a given soil condition the K_{Ic}-values practically are not clod size depend and characterize the soil. The averaged K_{Ic}-values for clod sizes are presented in Table 3.

DISCUSSION

Taking into account the physical meaning of each of the three computed soil parameters in Table 3, one can see that their values and trends of change agree with the transition from virgin to cultivated (soil 1), and from cultivated for 27 years with rather small clay content (soil 1) to cultivated for 45 years with high clay content (soil 2). According to Hadas and Wolf [6], clay content is 16 % of dry weight for soil 1, and 47 % of dry weight for soil 2.

Consideration of the first of these transitions, in Table 3, shows that, on the one hand, there is observed an increase in the cracking and the crack net connectedness in the clods, and also a volume share growth of the clods less then 80 mm. But on the other hand, there is an increase in critical stress-intensity factor and, hence, the crack growth resistance in the clods. The later can be attributed to cultivation induced soil compaction.

Consideration of the second mentioned transition, in Table 3, regarding crack net connectedness and clod volume share with a clod size less then 80 mm, shows the strong influence of clay content in suppressing soil cracking and fragmentation of large clods in soil 2, as compared to soil 1. And all that takes place in spite of the essentially larger duration of mechanized cultivation for soil 2. The growth of the critical stress-intensity factor for soil 2 as compared to soil 1 is also a natural manifestation of the simultaneous action of both the large duration of mechanized cultivation, leading to soil compaction, and the large clay content.

Table 3. Estimates of soil properties under investigation as affected by soil type and soil usage

Soil No.	Soil usage and duration of mechanized cultivation	Crack net connectedness	Clod volume share for size less then 80 mm	Crticial stress-intensity factor kPa m $^{1/2}$
1	Virgin soil	0.01	0.08	0.18 ± 0.02
	Cultivated for 27 yr	0.10	0.57	0.39 ± 0.07
2	Cultivated for 45 yr	0.35	0.25	3.6 ± 0.5

Table 4. Critical stress-intensity factor (kPa $m^{1/2}$) as dependent on soil type, soil usage and soil size

Soil No.	Soil usage and duration of mechanized cultivation	Clod size (mm)						
		2	5	10	20	40	70	80
1	Virgin soil	0.16 $\pm$ 0.05	0.20 $\pm$ 0.08	0.21 $\pm$ 0.09	0.19 $\pm$ 0.08	0.16 $\pm$ 0.06	0.16 $\pm$ 0.05	0.16 $\pm$ 0.05
	Cultivated for 27 yr	0.43 $\pm$ 0.18	0.49 $\pm$ 0.27	0.40 $\pm$ 0.20	0.31 $\pm$ 0.21	0.27 $\pm$ 0.18	0.36 $\pm$ 0.21	0.45 $\pm$ 0.25
2	Cultivated for 45 yr	3.1 $\pm$ 1.1	4.3 $\pm$ 1.9	4.0 $\pm$ 1.6	3.5 $\pm$ 1.5	2.8 $\pm$ 1.4	2.8 $\pm$ 1.4	3.8 $\pm$ 1.5

CONCLUSION

An approach to evaluate the crack net connectedness and the critical stress-intensity factor for air-dry soil was suggested, and founded on the basis of a theoretical estimate of soil clod strength, and on fitting literature data [6].

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